THE IMPORTANCE OF INVESTOR HETEROGENEITY AND FINANCIAL MARKET IMPERFECTIONS FOR THE BEHAVIOR OF ASSET PRICES

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by

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I. Introduction

Many features of asset prices and investment behavior have proven hard to explain using simple parametric versions of representative agent models estimated with aggregate consumption data. One of the most often cited difficulties is in reconciling the large spread between the mean return on stocks and bonds--the equity premium puzzle of Mehra and Prescott (1985). Not only is the predicted equity premium too low (assuming "moderate" levels of risk aversion), but the predicted risk-free rate is too high (for "plausible" rates of time preference). This latter puzzle is known as the "risk-free rate puzzle" [Weil (1989)]. Closely related is the fact that observed holdings of risk-free assets greatly exceed predicted holdings in models in which asset returns are taken as given and quantities are chosen to clear markets. The representative agent model also appears unable to explain the term structure of interest rates [Backus, Gregory, and Zin (1989)], or to match the second moments and correlation structure of asset returns. These pricing puzzles are to a large extent due to the fact that aggregate consumption is not very risky, which implies that a moderately risk-averse representative agent demands only a small premium to hold stocks over risk-free bonds, and has a low precautionary demand for savings. Representative agent models also are not designed to explain asset market phenomena such as the high concentration of stock ownership and the large volume of trade.

Several attempts have been made to reconcile the representative agent model with observed asset returns by considering deviations from the preferences considered by Mehra and Prescott (1985). For example, the first moments of returns can be more easily matched by assuming higher levels of risk aversion and a lower discount rate [Kocherlakota (1990)], by using preferences that exhibit habit persistence [Constantinides (1990)], or by using a nonexpected utility function [Epstein and Zin (1991)]. Although these modifications to the representative agent model have been successsful on some dimensions, they do not provide a completely satisfactory characterization of asset prices. For example, Heaton (1993) shows that although habit persistence models can explain the equity premium puzzle and risk-free rate puzzle, they do so at the cost of highly volatile short-term interest rates. Since the observed volatility of short-term interest rates is quite low, this presents an additional challenge to these models. More generally, Cochrane and Hansen (1992) show that matching first and second

moments simultaneously is likely to be difficult in many of these models because although asset return data implies highly volatile intertemporal marginal rates of substitution (IMRS), this volatility should not manifest itself as a volatile conditional mean of the IMRS.

An alternative approach is to relax the assumption of complete markets. Clearly there are many ways to do this, from explicitly modeling information problems to assuming a particular structure of transactions costs or institutional restrictions. The incomplete markets models that have been designed to address the quantitive issues raised by the standard asset pricing puzzles typically postulate a non-contractible source of risk, usually taken to be labor income risk. For instance, if individuals cannot fully insure against spells of unemployment due to moral hazard or other contractual problems, individual consumption will fluctuate with employment status. In addition agents are often assumed to face market frictions in the form of trading costs and borrowing constraints. These frictions break the link between individual and aggregate consumption, allowing for the possibility that individual consumption growth is more volatile than is aggregate consumption growth. This in turn affects predicted asset prices because in the presence of uninsurable risk individuals become more reluctant to hold risky assets and are more inclined to save. Furthermore, since agents make investment decisions based on net-of-transactions-costs returns, these costs also have a direct affect on equilibrium asset prices.

Structurally, many of the models with frictions that have been proposed to date fall into one of three broad categories:[1]

1. Models in which agents are *ex ante* identical and the equilibrium is characterized by no trade. Two period models of this type can be found in Heaton and Lucas (1992), Mankiw (1986), Scheinkman (1989) and Weil (1992). A multi-agent, infinite horizon model of this type has been developed by Constantinides and

[1] Due to space constraints and the difficulty of integrating two quite distinct modeling approaches, we largely neglect the important literature on the effects of information asymmetries on asset prices (e.g., the credit rationing model of Stiglitz and Weiss (1981) and the market microstructure literature as surveyed by Admati (1991) and Spatt (1991)).

Duffie (1994).

2. Models in which a continuum of agents trade securities to buffer their income shocks, but there are no aggregate shocks. In these models a stationary equilibrium can be characterized by a constant cross-sectional distribution of asset holdings and constant returns. Examples include Bewley (1984), Aiyagari (1992), Aiyagari and Gertler (1991), Clarida (1990), and Hugget (1991). Vayanos and Vila (1993) study a related continuous time life-cycle model.

3. Models in which two types of agents face aggregate as well as idiosyncratic income shocks, and trade over time in the asset markets to partially buffer the idiosyncratic shocks. Examples include Heaton and Lucas (1994), Lucas (1991), Marcet and Singleton (1991), Telmer (1993), and Telmer and Zin (1993). These can be further divided between models with and without transactions costs in asset markets.

Despite these differences across models in structure and in the assumed form of frictions, in this paper we make the case that a number of apparently robust implications can be drawn from these analyses, and that these results make the traditional asset pricing puzzles much less puzzling. This is due to the fact that in models with uninsurable idiosyncratic income shocks, predicted average asset returns are extremely sensitive to the presence of borrowing constraints, trading costs, the spread between borrowing and lending rates, and to the persistence of uninsurable income shocks. While consideration of these factors are shown to produce model estimates that are broadly consistent with many features of the data, many questions remain. For instance, for these models to be considered successful from a more mainstream finance perspective, the test will be whether including measures of income variability and other frictions will improve the explanatory power of more applications-oriented asset pricing models such as the APT. From a theoretical perspective, it remains to show that the implications of these models are robust to endogenizing the assumed market frictions.

The remainder of this paper is organized as follows: In Section II we present a series of theoretical models that illustrate the interactions and the potential magnitudes of the effects

of various forms of frictions, and separately survey some of the related literature that cannot be mapped directly into this framework. In Section III we discuss the growing body of empirical evidence that directly or indirectly bears on the importance of frictions for asset prices and asset market behavior. Section IV concludes.

II. Theoretical Models

In this section we present a series of incomplete market models in order to systematically show how various forms of market frictions can affect predicted asset prices. In particular, we investigate the effects of non-contractable shocks to individual income, the persistence and conditional volatility of these individual shocks, borrowing and short sales constraints, and transactions costs in asset markets.

A. Models with a Continuum of Agents, A Single Asset, and Undiversifiable Idiosyncratic Income Shocks

We begin with a simple model, similar to Huggett (1991), which illustrates the qualitative and quantitative effects of uninsurable idiosyncratic income shocks. We show that the predicted risk-free rate is highly sensitive to the degree to which borrowing is constrained, the quantity of outside debt, the persistence of idiosyncratic shocks, and, in some cases, to moderate trading costs in the bond market. These results are consistent with the findings of Constantinides and Duffie (1994), Huggett (1991), Heaton and Lucas (1994), Lucas (1991), Marcet and Singleton (1991) and Telmer (1993).

Let J index the set of agents in the economy and suppose that the utility function for j $\in$ J, over consumption $\{c_{j,t}$: $t=0,1,2, \ldots, T\}$ is given by:

$$U_{j,0} = E\left(\sum_{t=0}^{T} \beta^{t} \frac{c_{j,t}^{\ 1-\gamma}}{1-\gamma} | \mathscr{F}_0\right) \tag{1}$$

where γ is the coefficient of relative risk aversion. We use this utility function for expositional simplicity and because it is used extensively in the literature. Under the assumption of complete

markets, this utility function also implies that there is a representative agent with the same preferences defined over aggregate consumption $c_{a,t}$ where:

$$c_{a,t} = \sum_{j \in J} c_{j,t} \tag{2}$$

Suppose now that these agents live in a world in which aggregate income growth, $\lambda_t = c_{a,t}/c_{a,t-1}$, is distributed identically and independently over time. Under the assumption of complete markets and two equally likely states for λ_t, the price of a risk-free bond is given by:

$$p_b = \frac{\beta}{2}\left[(\lambda^1)^{-\gamma} + (\lambda^2)^{-\gamma}\right]. \tag{3}$$

Assuming that λ_t takes on the values $\lambda^1 = 1.054$ and $\lambda^2 = 0.983$,[2] and setting $\beta = 0.95$ and $\gamma = 1.5$, the implied price of the bond is 0.93, which yields a risk-free return of 7.9%. For comparison, the average annual real return on Treasury securities over the period 1889-1978 was 0.8%. Of course the bond return could be lowered simply by making β larger. For example setting $\beta = 0.98$ implies that the bond return is 4.6%. However once stock returns are added to the model, β cannot be chosen to solely fit the low bond return.

We will see that adding uninsurable income risk to this model can (but need not) generate a much lower risk-free rate without any change in the utility parameters. With incomplete securities markets, individual consumers are unable to completely diversify away idiosyncratic income risk. For the utility specifications commonly assumed, this additional risk increases the precautionary demand for savings[3] and lowers the risk free interest rate. The magnitude of this effect depends on the size and persistence of the idiosyncratic income shocks, and on the extent

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[2] These values are chosen so that aggregate consumption growth has a mean of 1.8% and a standard deviation of 3.57%, which matches the mean and standard deviation of annual consumption growth from 1889 to 1978 as reported by Mehra and Prescott (1985).

[3] This occurs because the marginal utility of consumption is convex in consumption. See Kimball (1990) for a general discussion of precautionary savings.

to which agents can use the available asset markets to share risk.

To formalize this idea, let J be a continuum of measure one (the number of agents is normalized to one), and suppose that individual agents are endowed with consumption $\{e_{j,t}$: $t=0,1,2, \ldots, T\}$ where $e_{j,t}=\eta_{j,t}c_{a,t}$. To begin with we assume that $T=1$, that $\eta_{j,0}=1$ for all j, and that agents at time 0 can only trade a risk-free bond. Since agents are *ex ante* identical, there is no trading at time 0 and the time 0 price of the bond is given by:

$$p_b = \beta E\left[(\lambda_1 \eta_{j,1})^{-\gamma} | \mathcal{F}_0\right] \tag{4}$$

Suppose that for each j, $\eta_{j,1}$ can take on the two values with equal probability, $\eta^1=0.75$ and $\eta^2=1.25$, and that $\eta_{j,1}$ is independently distributed across agents. This distribution of $\eta_{j,1}$ matches the variance of the innovation of annual household income in the PSID.[4] Given this level of individual income or endowment uncertainty, a two-period model implies a risk-free bond return of -4.3%. The presence of substantial income variability at the individual level induces a large precautionary savings motive which implies a very low equilibrium rate of return on savings. This only occurs, however, because the agents are assumed to "eat" all of their income variability.[5]

Suppose now that T = ∞, and that agents can trade in one period bonds at t=1,2, ... to partially offset the idiosyncratic income shocks. Note that each agent would like to build a *buffer stock* of savings in order to more effectively smooth consumption over time. At each t, agent j faces the budget constraint:

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[4] See Heaton and Lucas (1994) for details.

[5] Mankiw (1986) first exposits this idea in a two period model.

$$c_{j,t} + p_{b,t}b_{j,t+1} \leq e_{j,t} + b_{j,t} \tag{5}$$

where $c_{j,t}$ is individual j's consumption at time t, $p_{b,t}$ is the price of the bond at time t and $b_{j,t+1}$ is the face value of individual j's bond purchases or sales at time t.

To eliminate Ponzi schemes it is necessary to somehow restrict borrowing.[6] In contrast to representative agent models, with incomplete markets the implied asset prices may be quite sensitive to the assumed form of this restriction. One possibility here is to exogenously impose a borrowing constraint on each individual, which in some circumstances is equivalent to imposing a present value budget constraint [Aiyagari (1992)]. We employ the following constraint:

$$b_{j,t} \geq -K_j c_{a,t}. \tag{6}$$

K_j in (6) is nonnegative, implying that agent j can borrow up to a maximum percentage of the aggregate endowment. This assumption accomodates an equilibrium with stationary prices in a growing economy. As shown below, the opportunities for consumption smoothing--and hence the equilibrium price of bonds in this economy--will depend on the cross-sectional distribution of bond holdings and the proportion of the population who are constrained. Since it is not clear how to set K_j, we experiment with a range of values for this constraint.

For the infinite horizon version of the model, we continue to assume that for each agent $\eta_{j,t} = e_{j,t}/c_{a,t}$ takes on the two values, $\eta^1=0.75$ and $\eta^2=1.25$, but simplify by assuming that the

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[6] Borrowing constraints also may arise endogenously in models with asymmetric information. For example, see Stiglitz and Weiss (1987).

aggregate endowment grows at a constant rate of 1.8% per year.[7] The equilibrium then can be characterized by solving for the constant bond price and stationary distribution of bond holdings that clears the bond market. We will see that the equilibrium interest rate is significantly affected by the assumed persistence of the idiosyncratic shocks. Data suggests that shocks to relative labor income are fairly persistent over time. For instance, Heaton and Lucas (1994) estimate a first-order autoregression for $\eta_{i,t}$ and find the first-order autocorrelation to be 0.52. To capture this persistence, we assume that $Prob\,(\eta_{j,t+1}=\eta^1 | \eta_{j,t}=\eta^1)=0.74$ and $Prob(\eta_{j,t+1}=\eta^2 | \eta_{j,t}=\eta^2)=0.74$.[8] We also initially assume that there is no outside supply of bonds in the economy so that market clearing implies that $\sum_{j\in J} b_{j,t}=0$.

To quantify the effects of these various frictions the model is solved numerically. An approximate equilibrium is found by specifying a finite grid of possible values for individual bond holdings. Given a bond price, p_b, for each initial level of bond holdings and realized income state an approximate bond purchase rule is found by minimizing the Euler equation errors implied by the discrete approximation. Given these bond holding rules, a stationary distribution of bond holdings is calculated using the method described in Hugget (1991). The price of the bond is then adjusted to clear markets.[9]

We first consider seveval settings for the proportional borrowing constraint K_j. For maximum borrowing ranging from 10% to 50% of per capita income, Table I reports the

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[7] Any assumed change in the aggregate endowment over time would imply a time-varying distribution of asset holdings and bond returns, complicating even numerical solution. Recently, however, den Haan (1994) has developed a numerical technique for approximating the equilibrium in models with aggregate shocks and large (finite) numbers of agents.

[8] These transition probabilities were found by fitting a two-state Markov chain to the first-order autoregressive model for $\eta_{j,t}$ using the methods of Tauchen and Hussey (1991). See Heaton and Lucas (1994) for details.

[9] This solution method is similar to that used by Hugget (1991), and is also related to the *auctioneer algorithm* of Lucas (1994).

equilibrium interest rate along with the percentage of the population that in equilibrium is constrained.

Table I: The risk-free interest rate and percentage of constrained borrowers in the population as a function of borrowing constraints

K_j	Risk-Free Rate (%)	% of Population Constrained
0.1	-3.2	40.4
0.2	1.0	32.5
0.3	2.9	24.5
0.4	4.2	21.5
0.5	5.1	16.8
persistence from the PSID, no outside bonds		

Notice that as K_j increases the risk-free rate rises sharply and the percent of the population at a corner declines. This occurs because with increased borrowing opportunities agents are better able to smooth their consumption over time and hence their precautionary demand for savings declines. As a result, a higher risk-free interest rate is needed to clear the bond market. For the model to predict a risk free rate as low as the historical average of nearly zero, a substantial fraction of agents in the economy must be constrained at any point in time. For example, producing a real rate of 1% requires that in equilibrium just over 30% of the population is against the borrowing constraint. One might view this as an unrealisticly large number of constrained agents, especially considering the fact that a typical household has at least some savings that can be used for consumption smoothing. Notice that these results are based on the assumption that bonds are in zero net supply, so that household savings is on average zero.

To examine the sensitivity of predicted returns to the presence of positive amounts of net savings, we introduce an outside supply of bonds. These bonds can be interpreted as "trees" that

produce consumption at a fixed rate given by the equilibrium interest rate in the economy. The holdings of liquid, low risk securities in the U.S. economy is skewed towards upper income households, and is relatively low for the median family [Avery Elliehausen and Kennickell (1988)]. For example, in the 1984 PSID the median household has 3.3 months of income in liquid securities. To reflect this level of median savings, we solve the model for various levels of borrowing constraints under the assumption that the median holdings of bonds relative to income is 0.27, which is equivalent to 3.3 months of income in this annual model.

Table II: The risk-free interest rate and percentage of constrained borrowers in the population as a function of borrowing constraints

K_j	Risk Free Rate (%)	% Popuation Constrained
0.1	4.4	20.2
0.2	5.3	14.8
0.3	5.8	13.8
0.4	6.3	11.5
0.5	6.8	10.0
persistence from the PSID, outside bonds = .27 median income		

The calculations in Table II demonstrate that with the addition of a relatively small amount of liquid assets, the bond return is much higher for each value of K_j. For example, when $K_j = 0.1$ the predicted return on savings is over 7% higher than in the case in which bonds are assumed to be in zero net supply, and much closer to the complete markets benchmark. As the borrowing constraint is relaxed, the predicted return with and without outside bonds tends to converge since few agents are constrained.

The high sensitivity of the predicted risk-free rate to borrowing constraints and the agregate supply of liquid assets appears to be robust to model specification, as similar results are reported by Lucas (1991), Marcet and Singleton (1991), and Telmer (1993) in the context of two agent models with aggregate shocks as well as idiosyncratic shocks. These results are

also consistent with Aiyagari (1992), who looks at a setting similar to this one but with an endogenous capital stock. He finds that for low levels of risk aversion and small amounts of persistence in the idiosyncratic shocks, the aggregate savings rate and the return on savings is very close to that predicted in the complete markets benchmark. In his model, this implies that precautionary savings accounts for a small percentage of observed savings.

As shown quite generally by Constantinides and Duffie (1994), the persistence of idiosyncratic income shocks can significantly lower predicted rates of return relative to the complete markets case *even if borrowing constraints never bind.* This is because persistent shocks have a wealth effect that increases the correlation between income and planned consumption, and hence increases the volatility of consumption.[10] To quantify the effect of varying the level of persistence on the predictions of this model, we consider the implications of assuming different values of the autoregressive parameter in the underlying first-order autoregression for $\eta_{i,t}$. As above we use an approximating Markov chain model to approximate the autoregressive parameters, here assumed to be 0, 0.5 (which is similar to that found in cross-sectional data), and 0.8. The unconditional variance of the shocks is fixed across experiments.[11] To better approximate the model with first-order autocorrelation of 0.8, we expand the state space to 4 states. The new state space for this case is given in Table III.

[10] Although we do not consider it here, the possibility of a very bad income realization can also lower the riskfree rate. For example, Carroll (1992) stresses the importance of bad realizations for a buffer stock model of savings, and Rietz (1988) makes the point for aggregate consumption.

[11] In these finite state space approximations, the question arises of whether it is better to fix the unconditional variance of consumption, or to fix the state space of consumption and alter only the transition matrix to change the autocorrelation. Although the first alternative seems more natural, it has the problem that the support of the underlying state space changes across cases, leaving open the possibility that the change in predicted rates is more a response to the possibility of a very bad income realization at any point in time rather than to increased persistence per se.

Table III: Income States

State Number	$\eta_{i,t}$
1	0.64
2	0.85
3	1.10
4	1.46

Table IV reports equilibrium interest rates and the percentage of constrained agents for the three autoregessive parameters and two values of the borrowing constraint. This highlights an interesting phenomena that as far as we know has not been explored in the literature--the interaction between the persistence of shocks and the severity of borrowing constraints is highly non-linear. For the assumed model parameters and K_j = .5 adding

Table IV: The risk-free interest rate and percentage of constrained borrowers in the population as a function of borrowing constraints and the persistence of shocks

ρ	K_j	Risk Free Rate (%)	% Popuation Constrained
0.0	0.1	-4.7	42.9
0.0	0.5	6.8	7.8
0.5	0.1	-2.7	40.4
0.5	0.5	5.1	16.8
0.8	0.1	1.1	48.0
0.8	0.5	5.8	8.2
no outside bonds			

persistence decreases and then increases the risk-free rate, but with K_j = .1 adding

persistence increases the risk-free rate. This non-linear behavior can be interpreted as follows. In the absence of borrowing constraints, increasing the persistence of shocks should unambiguously decrease the risk-free rate because the wealth effect increases the precautionary demand for savings. With a relatively tight borrowing constraint, however, increasing persistence can increase the fraction of constrained agents because bad shocks tend to be followed by more bad shocks. Offsetting this is the fact that due to the wealth effect, agents want to trade less when shocks are more persistent, so for high levels of persistence the borrowing constraint becomes less relevant. The interaction of these two effects creates the ambiguous situation in Table IV, in which seeing a higher percentage of constrained agents no longer guarantees a lower equilibrium rate. Notice that these findings also weaken the risk-free rate puzzle, in that a wide range of interest rates are consistent with moderate changes in the assumed persistence of income shocks.

We have seen that even with moderate persistence in idiosyncratic shocks and limited ability to borrow, this model predicts that agents can fairly effectively smooth consumption by trading in the bond market. As a result, predicted asset prices look much like those with complete markets unless a severe borrowing constraint is assumed. One factor that could significantly alter predicted trading volume and therefore the extent of consumption smoothing is transactions costs. For instance, borrowing involves expenses such as credit checks, points, application fees, and verification of collateral value. These costs tend to discourage borrowing in a way that is different than for an absolute restriction on the amount of borrowing.

To examine the effects of transactions costs on predicted asset prices, we consider two simple extensions of the above model in which transactions costs are quadratic. In the first we suppose that agents pay transactions costs only when they borrow. Although we do not model this from first principles, it is intended to capture the observed spread between the borrowing and lending rate for low-risk consumption loans. In this case the cost function is one-sided and takes on the form:

$$\Omega\ (b_{i,t+1};p_b) = \omega_t[\min(0,b_{i,t+1})p_b]^2 \tag{7}$$

where ω_t controls the size of the cost. Since the economy is assumed to grow over time, the aggregate value of the bonds being traded also grows. To induce stationarity in the cost function, we assume that $\omega_t = \omega/c^a_t$. Notice that in (7) the transactions costs are paid every period, even if the outstanding amount of debt remains the same. This is consistent with the idea that the costs are part of the spread between borrowing and lending rates. Alternatively, trading costs may also be incurred when bond are purchased. These costs could be brokerage fees, bank fees or other costs incurred in adjusted a bond portfolio. The cost function in this case is symmetric or borrowers and lenders, and given by:

$$\Omega\ (b_{i,t+1},b_{i,t};p_b) = \omega_t(b_{i,t+1}p_b)^2\ . \tag{8}$$

Tables V and VI report the equilibrium lending rates implied by these cost specifications for an ω ranging from 0.0 to 0.2. To provide an interpretable metric for the size of these costs, we also report the expected total incurred transactions costs as a percentage of the value of bonds traded. The borrowing constraint, K_j, is fixed at 0.5 since in the above simulations it was shown to bind fairly infrequently.

Table V: The risk-free interest rate as a function of trading costs; one-sided costs.

ω	Average Costs (%)	Risk-free Rate (%)	% Population Constrained
0.00	0.0	5.1	16.8
0.05	0.8	4.3	13.8
0.10	1.4	3.5	7.1
0.15	1.9	2.6	0.0
0.20	2.2	1.8	0.0
no outside bonds, K_j=0.5			

Table VI: The risk-free interest rate as a function of trading costs; two-sided costs.

ω	Average Costs (%)	Risk-free Rate (%)	% Population Constrained
0.0	0.0	5.1	16.8
0.05	1.4	6.5	9.6
0.10	2.3	6.5	0.0
0.15	2.8	6.2	0.0
0.20	3.1	6.0	0.0
no outside bonds, K_j=0.5			

Table V shows that as transactions costs rise agents choose to borrow less, so that the borrowing constraint binds less frequently. Even so the risk-free rate falls, which can be attributed in part to increased consumption variability (and therefore to an increased precautionary demand), and in part to the fact that market rates adjust to keep the net-of-transactions costs rates at an equilibrium level. To elaborate on this second point, note that the predicted market rates in Tables V and VI are quite different, even for similar levels of average transactions costs. This is due to the fact that the effect of transactions costs on

interst rates depends critically on the assumed incidence of those costs.[12] In fact the implied average effective marginal borrowing and lending rates turn out to be very close when average transactions costs are the same. For example consider the case in Tables V and IV in which the average percentage transactions costs are both 1.4%. With one-sided costs the lender receives a return of 3.5%, and on the margin the borrower pays 3.5% + 4(1.4%) = 9.1%.[13] With two-sided costs, the lender receives a marginal return of 6.5 - 2(1.4) = 3.6%, while the borrower pays 6.4 + 2(1.4) = 9.2%.

How would observed market interest rates be affected by other cost structures? A quadratic form was assumed here because it greatly simplifies the computation of an equilibrium. If instead costs were assumed to be proportional, we would expect qualitatively similar results since in both cases the volume of trade is reduced by the presence of the costs. Proportional and fixed costs of transacting in a similar model have been considered by Aiyagari and Gertler (1991).

B. Relative Rates of Return Without Aggregate Risk

So far we have assumed that the only tradable asset is a one period risk-free bond. In reality households have access to many securities that can be used for consumption smoothing, and market frictions may have a significant impact on the *relative rates of return* on these different securities. As in the bond example just discussed, the inclusion of differential trading costs across assets has a direct affect on predicted asset prices. This possibility is explored by Amihud and Mendelson (1986) in a model with an exogenous distribution of holding periods, and by Vayanos and Vila (1992) in a continuous time overlapping generations life-cycle model.

Taking an approach more similar to that of the previous section, Aiyagari and Gertler (1991) study differential transactions costs in an economy with a continuum of agents, uninsurable idiosyncratic income risk, and no aggregate uncertainty. Transactions in the stock market are subject to proportional or fixed costs but bond market transactions are assumed to be costless. A borrowing constraint limits the extent to which agents can

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[12] In Heaton and Lucas (1994), we find similar results in a two agent model with aggregate and individual shocks.

[13] In this calculation the transactions cost is multiplied by four because costs are quadratic, and because the lender pays none of the cost.

substitute towards trading in the low-cost bond market. The return differential between stocks and bonds is fixed at 3%, and the model is solved for a quantity of government bonds and a tax policy that supports these differential returns. They conclude that although the model does a fairly good job of explaining the relative transactions velocities of stocks and bonds and the ratio of stocks to income, it cannot explain the large observed holdings of liquid assets by U.S. households. This is equivalent to the conclusion above that with enough low-cost trading opportunities, models with idiosyncratic income shocks cannot explain the low risk-free rate.

C. Models with Aggregate Risk, Multiple Assets, and Undiversifiable Idiosyncratic Income Shocks

The models in Section II.B demonstrate that some part of the differential between stock and bond returns is likely to be attributable to differential trading costs. Another potentially important factor, and the one that has been central to the analysis of representative consumer models, is risk differentials across securities. To evaluate the effect of risk on return, aggregate as well as individual income shocks must be taken into account. In models with both sources of risk, predicted returns appear to be sensitive to two factors we have yet to consider: the assumed interaction between borrowing and short sales constraints, and the conditional variance of the idiosyncratic income shocks. In this subsection we explain how these factors influence predicted return differentials between stocks and bonds, and describe calibration results from a model that attempts to quantify the importance of these and other frictions.

i. The influence of borrowing and short sales constraints.

For concreteness, suppose now that individuals are faced both with aggregate and non-contractable idiosyncratic income shocks, and can trade in a one period bond and a stock. A share of stock entitles its holder to a flow of dividends over time. In this case the budget constraint (5) becomes:

$$c_{j,t} + p_{b,t} b_{j,t+1} + p_{s,t} s_{j,t+1} \leq e_{j,t} + b_{j,t} + s_{j,t}(p_{s,t} + d_t) \tag{9}$$

In (9), $p_{s,t}$ is the price of the stock at time t, $s_{j,t+1}$ is the amount of stock held from time t to time t+1 and d_t is the dividend paid at time t.

As in single asset models, some type of wealth constraint must be imposed on each

agent. In a world with several securities and uncertainty there are several ways in which this could be accomplished. One way is to impose separate constraints on borrowing and on the short sale of stock, as in Aiyagari and Gertler (1991), Heaton and Lucas (1994), Lucas (1992), and Marcet and Singleton (1992). In the models developed above, this would mean augmenting (7) or (8) with a similar constraint in the stock market. The assumption of separate constraints can be motivated by the observation that in decentralized markets, the cost of enforcing contracts is likely to vary across markets. With separate constraints, the number of constrained agents tends to vary across markets. This affects relative rates of return because the price of each security only reflects the marginal rates of substitution of unconstrained agents.

An alternative with different relative price implications is to restrict borrowing using what Cochrane and Hansen (1992) call a "market wealth constraint." This has been used, for example, by He and Modest (1991), Hindy (1992), Luttmer (1994), Zeldes (1989) and others. Under this constraint, the value of an individual's current portfolio must be greater than some minimum:

$$p_{b,t}b_{j,t+1} + p_{s,t}s_{j,t+1} \geq -K_j \tag{10}$$

The Euler equations for the optimization of (1) subject to (9) and (10) imply that:

$$E\left[\beta\left(\frac{c_{j,t+1}}{c_{j,t}}\right)^{-\gamma}\left(r_{s,t+1} - r_{b,t}\right)|\mathcal{F}_t\right] = 0 \ . \tag{11}$$

Since under (10) even a wealth-constrained agent can adjust the relative amount invested in each asset, relative rates of return are determined by a standard consumption-based CAPM (CCAPM), but with individual consumption in the place of aggregate consumption. As we have seen, when markets are incomplete individual consumption is potentially more volatile than is aggregate consumption, so a model in which (11) holds can in principle produce a realistic equity premium.

It is also possible, however, to construct models with a market wealth constraint which clearly will not help to resolve the equity premium puzzle. For instance even with

incomplete markets, if preferences are quadratic then linearity of the marginal utility of consumption implies a CCAPM for relative rates of return based upon aggregate consumption.[14] In models with a more general utility function in which securities span aggregate and individual uncertainty and with a market wealth constraint, it is possible to show that the intertemporal marginal rate of substitution (IMRS) based on aggregate consumption also correctly prices excess returns.[15]

In sum, the extent to which borrowing and short sales constraints can be expected to influence the predicted equity premium depends on the ability of agents to smooth their consumption over time, which in turn depends on the details of the assumed market structure. When agents can effectively smooth their consumption over time using the available asset markets, we saw above that the average level of interest rates will be similar to the complete markets case. Not surprisingly, with sufficient smoothing opportunities relative rates of return will also be similar to the complete markets case. Finally, it should be noted that in models without an aggregation theorem, directly testing (11) is impractical because of the increased data requirements imposed by heterogeneity.

ii. The conditional volatility of idiosyncratic shocks.

When market incompleteness results in a significant reduction in the ability of agents to smooth consumption over time, the average level of returns on all securities is reduced due to a precautionary effect as discussed in Section II.A. To the extent that the lack of smoothing results in a change in the *covariance* between individual IMRS's and excess returns, then predicted excess returns will also be affected. In particular, an increase in the variance of individual shocks in downturns implies that agents view claims to aggregate risk as being very risky. This effect of conditional volatility with incomplete markets was first discussed by Mankiw (1986) and formalized and extended to a more general setting by Constantinides and Duffie (1994).

Constantinides and Duffie derive a closed form solution for asset returns when shocks to individual income are constructed to be effectively permanent so that in equilibrium there is no trade. If the shocks to individual income are assumed to be independent of the

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[14] Hindy and Huang (1993) exploit this idea to derive a CAPM under the assumption of quadratic preferences. They also consider a situation in which the securities have different collateral value.

[15] This is demonstrated, for one, by Cochrane and Hansen (1992).

aggregate state, then the equity premium predicted by the model is the same as in the complete markets case. However when the conditional variance of the individual shocks is countercyclical, then the model predicts a higher equity premium since the IMRS of each individual is negatively correlated with the payoffs on stocks. As discussed as in Section III below, there is little empirical evidence to support a significant effect of this type.

iii. Results from calibrated models.

As the preceding sections make clear, whether these models with frictions will be able to satisfactorily explain asset return differentials will depend, among other things, on the observed magnitudes of the frictions. To illustrate how the relative return on stocks and bonds is affected by the size and structure of trading costs and by the joint statistical process governing aggregate and individual income growth, here we report calibration results from a model [Heaton and Lucas (1994)] that incorporates aggregate risk, the possibility of persistence in the income process, and a risky and risk-free asset. In order to compute an equilibrium in this more complicated setting, we assume a two-agent economy. It appears that going from a model with a continuum of agents to one with only two types of agents should not significantly affect predicted prices.[16]

Agents in this two-agent model face the same optimization problem as in (1) with the budget constraint (9). Costs of trading in the bond market are quadratic as in (7), and costs of trading in the stock market are quadratic in the value of the stock traded. Aggregate risk is reflected in labor income and in dividends. As in the model of Section II.A, individuals also face uninsurable labor income risk. The model is calibrated using a statistical model of individual income that reflects the size and persistence of idiosyncratic shocks, based upon evidence from the Panel Study of Income Dynamics (PSID). The time series properties of aggregate income and dividends are also estimated using the National Income and Product Accounts. Since little direct evidence is available on the magnitude of transactions costs, the model is solved for a plausible range of assumed one-sided and two-sided trading costs.

Transactions costs have what can be described as a direct and an indirect affect on asset prices in this model. As in the models in Section II.B, the direct affect is due to the fact that

[16] This is evidenced by the similarity of the results in Section II.A to those from the two agent models of Lucas (1991), Marcet and Singleton (1991), and Telmer (1993), and also by den Haan (1994), who constructs a model with a variable number of agents and shows that the number of agents has little affect on mean returns although it can alter unconditional variances.

the equilibrium gross rates of return on securities is altered by the presence of transactions costs, even in a world where there is no aggregate income or dividend risk. Because of this direct effect, a substantial equity premium can be generated by identifying the Treasury bill rate with the lending rate and by assuming that only borrowers pay a transactions cost in the bond market, while both parties pay transactions costs in the stock market. The indirect affect of transactions costs is that they discourage the use of asset markets to offset transitory income shocks, so that consumption becomes more variable. The increased consumption volatility reduces the tolerance for aggregate uncertainty, which tends to increase the equity premium. It also lowers average returns on both assets by increasing the precautionary demand for savings.

The main results of this calibration exercise are the following: Agents strongly substitute towards trading in the lower-cost market, so that for transactions costs to affect prices they must be simultaneously present in both markets. With a moderate wedge between the borrowing and lending rate or a frequently binding borrowing constraint, the model can generate a high equity premium and a bond return that is close to the observed return on U.S. Treasury securities. This is due to the fact that the net-of-transactions-costs rate of return on all securities falls in equilibrium due to the precautionary effect, and therefore the returns on Treasury bills fall since they are not subject to transactions costs when purchased.[17] The observed return on stocks is not substantially affected, although the net-of-transactions costs return on stocks does fall.[18] Breaking down the effects on returns into a direct and indirect effect, the direct effect accounts for a large portion of the predicted deviation from the representative agent model. Also although in principle the persistence and conditional volatility of individual income shocks could justify the observed rates of return under the assumption of lower levels of transactions costs, these income effects are hard to find using the PSID, and hence have little affect on the predicted returns in this model. Of course, this may be due to measurement error and the short time series in the PSID, rather than a true absence of these effects in the market.

C. Models with Frictions in Goods Markets

The link between aggregate consumption and equilibrium asset prices also appears to be

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[17] Vayanos and Vila (1993) also show in a life cycle savings model that the effect of transactions costs is to reduce the rate of return on the security not subject to transactions costs.

[18] The model of Aiyagari and Gertler (1991) also captures this affect of transactions costs.

easily broken by the assumption that consumption is costly to adjust. The basic idea, as exposited by Grossman and Laroque (1990), is that adjustment costs in consumption produce a smoother consumption stream than would be observed in a frictionless setting. They consider a partial equilibrium, continuous-time asset pricing model in which ownership of a durable good provides a flow of consumption services. Because of adjustment costs, agents optimally follow an s-S policy under which consumption changes only after a sufficiently large change in wealth, but trading in asset markets is costless. As a result the value of the risky stocks fluctuates with the aggregate endowment but not with consumption. Marshall (1992) demonstrates that this intuition carries over in an equilibrium model, and shows by calibration that even small adjustment costs can be sufficient to decouple consumption and asset prices. Lynch (1994) makes the related point that if fixed costs lead to staggered and relatively infrequent consumption and investment decisions across individuals, the correlation between contemporaneous asset prices and aggregate consumption will be small, and that asset prices will reflect lagged as well as contemporaneous consumption.

As in models with asset market frictions, the question arises of whether adjustment costs are large enough to have a measurable influence, particularly for non-durables. Both Marshall (1992) and Lynch (1994) argue that small costs are sufficient to significantly affect price predictions. From an empirical asset-pricing perspective, these models represent a negative result in that is not clear what to substitute for consumption to try to explain the behavior of asset prices. Nevertheless, these models are useful in that they provide another intuitively plausible mechanism for why consumption-based models may fail to explain asset prices.

D. When Frictions Have Little Effect

So far we have focussed on models in which frictions can significantly influence asset prices, but this need not be the case. Whether or not transactions costs matter is related to the strength of the assumed motive for trade. In the models discussed above, agents trade frequently even in the presence of transactions costs in order to offset relatively large idiosyncratic income shocks. In contrast, Constantinides (1986) considers a model in which agents can engage in costly trading in order to rebalance their portfolios. He finds that even small transactions costs cause agents to trade only very infrequently. As a result, few transactions costs are incurred and they have little affect on equilibrium asset prices. This is consistent with the finding of Lucas (1991) and Heaton and Lucas (1994) that if agents can choose between smoothing consumption by selling stocks or bonds, they use the instrument with the lowest trading cost with little regard for portfolio balance. These results highlight the importance of endogenizing the holding periods in models with transactions costs, since the impact of the costs is highly sensitive to the

frequency of trade.

III. Empirical Evidence on Market Frictions

As we have seen, explicitly modeling frictions generates a wealth of testable predictions and has the potential to resolve several of the more persistent asset pricing puzzles. The downside to this approach is that it greatly complicates empirical testing. The added complexity is largely due to the absence of aggregation theorems, which in the past have served to simplify data requirements. In particular, heterogeneity necessitates estimating individual as well as aggregate income or consumption processes, determining from cross-sectional data which agents are marginal and which are inframarginal, and taking a stand on how frequently agents must trade.

There is also uncertainty about the form and size of transactions costs. For instance, to what extent are costs fixed versus variable, and how important are borrowing or short sales constraints? The bid/ask spread is probably the easiest to measure proportional cost, but even here there are problems of data availability and interpretation.[19] Because of asymmetric information or thin markets, in some stocks even small trades can have a significant price impact, which can be viewed as another type of transactions costs. Another key issue is the magnitude of fixed costs that cannot be precisely measured, such as the cost of learning to evaluate the risk and return of various investment strategies.

Despite these complications, researchers have begun to put together a clearer picture of the nature and importance of heterogeneity, and the relation between transactions costs and measured returns. In the remainder of this section we summarize these findings and highlight issues that will require further research.

A. Indirect Evidence on Transactions Costs

Recent contributions to the volatility bounds literature provide indirect evidence on the potential importance of transactions costs to asset pricing models while avoiding many of the above-mentioned problems. Hansen and Jagannathan (1991) introduce an innovative method to quickly ascertain whether a frictionless asset pricing model is potentially consistent with the data.

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[19] Dealers may be able to trade inside the spread, suggesting that it overstate costs. At the same time individuals incur other expenses such as brokerage fees, suggesting that the spread understates costs.

They construct a region in mean-variance space in which the IMRS must lie. These volatility bounds are determined by the statistical properties of asset returns and are model-independent. The fact that the estimated IMRS for the representative agent model lies outside the estimated volatility bounds is consistent with tests of the Euler equations or "pricing errors" implied by these models.[20] This is often described by saying that with moderate risk aversion the IMRS implied by aggregate consumption data is not sufficiently volatile to be consistent with the equity premium.

How robust are these volatility bounds to the presence of frictions in asset markets? In the pricing models with frictions described above, rejection by volatility bounds tests is less likely than for representative consumer models because the estimated IMRS is typically more volatile. However Cochrane and Hansen (1992) show that in some contexts this can create additional puzzles. For example, in a model with borrowing constraints alone they use the volatility of the IMRS based on aggregate data to impose an additional lower bound on the IMRS of unconstrained agents. The volatility bound for the IMRS of unconstrained agents that takes this into account is quite high and may be implausible without the inclusion of other market frictions. When viewed as bounds on the volatility of the IMRS of a representative agent, the presence of market frictions can substantially weaken the volatility bounds [He and Modest (1993) and Luttmer (1994)]. Intuitively, a high equity premium can coexist with low variability of the IMRS because transactions costs make it less attractive for investors to adjust their portfolios to take advantage of the relatively high returns in the stock market.

To measure the importance of this effect, He and Modest (1993) and Luttmer (1994) extend Hansen and Jagannathan's framework to incorporate proportional trading costs and short sales constraints. Luttmer provides an aggregation result for a certain class of models with frictions.[21] Under the assumption of moderate trading costs (.5 percent for NYSE stocks and .125 percent for bonds), he finds that the IMRS no longer falls outside the weaker volatility bounds when estimated using quarterly or monthly aggregate consumption data and a power utility function with moderate risk aversion. Thus even small transactions costs have the potential to explain the rejection of the standard model.[22] It is important to note, however, that

[20] Hansen, Heaton and Luttmer (1994) provide a statistical methodology to conduct formal statistical inference and testing using the volatility bounds of Hansen and Jagannathan (1991), He and Modest (1993) and Luttmer (1994).

[21] His development relies on some of the analysis of Jouini and Kallal (1991).

[22] For a further discussion of these and related issues, see Hansen and Cochrane (1992).

implicit in this analysis is a holding period that corresponds to the data frequency of monthly or quarterly intervals. As in Constantinides (1986), for longer assumed holding periods transactions costs are incurred less frequently and hence have less effect on the volatility bounds.

B. Direct Evidence on Transactions Costs

There is a small but growing body of direct evidence that measured transactions costs have explanatory power for asset prices. This line of research is of particular importance because of the ambiguity in the theoretical literature on the magnitude of the predicted effects. To date surprisingly few authors have taken this approach, perhaps because of the difficulty in obtaining reliable transactions cost data as discussed above.

To generate testable predictions about the effect of the bid/ask spread on asset returns, Amihud and Mendelson (1986) consider a model with stochastic liquidity shocks in which the mean time to portfolio liquidation varies across investor types. Investors face an (exogenously given) bid-ask spread which varies across a set of (risk-free) assets in positive net supply. In equilibrium, investors with different horizons concentrate their portfolio holdings in certain assets, since a natural clientele is created by the fact that the return on high spread assets increases with the holding period. At the same time, the equilibrium promised return on high-spread assets is higher to compensate investors for anticipated transactions costs. To test for the predicted relation between asset prices and the bid-ask spread, they estimate several variants of the standard linear asset pricing model and apply it to NYSE data over the period 1961-80. Using 49 portfolios presorted by beta and spread, they regress the annual excess return on each portfolio on its beta and spread. As hypothesized, excess returns are found to increase in beta and also in the spread. To control for the fact that the bid-ask spread is highly correlated with firm size and that firm size has predictive power for asset returns, a size variable was also added to the regression. They find that the bid-ask spread and beta continue to have explanatory power for excess returns but size does not.

While these results appear to strongly support the hypothesis that transactions costs have a significant and predictable influence on asset prices, the results must be interpreted with some caution. Perhaps most problematic is that the empirical specification tested is unlikely to correspond to any equilibrium model. For instance, although the theory suggests that no investor would hold the market portfolio, the CAPM is used to measure priced risk. Presumably if risk aversion were explicitly taken into account, any correlation between realized liquidations and an appropriate measure of wealth would also affect the relation between predicted returns and spreads.

In a more recent paper on yield differentials in the Treasury securities market, Amihud and Mendelson (1991) provide additional evidence on the importance of transactions costs. This analysis avoids the above complications due to risk by comparing returns on Treasury bills and Treasury notes with only one remaining payment. Since both securities have a single certain payment at a known future date, in the absence of differential transactions costs or tax treatment,[23] the yield on a bill and a bond maturing on approximately the same date should be identical. They find, however, that annualized bond yields exceed similar bill yields by about .4 percent, with the yield differential decreasing with maturity. Since in the absence of transactions costs this would lead to easily exploited arbitrage opportunities, they test whether observable differences in transactions costs (bid-ask spreads, brokerage fees, and dealer fees) are large enough to preclude arbitrage. They find that for typical levels of these costs, traders on average would not profit by shorting bills and buying bonds, but investors who plan to hold to maturity appear to be better off buying bonds instead of bills.

The finding that the yield differential between bills and bonds is large for short maturities and decreases with maturity suggests the presence of fixed transactions costs, as do the observed institutional arrangements. Since even small fixed costs can lead to large differences in annualized returns for short maturity or frequently traded assets, this underscores the difficulty of directly relating yield differentials to transactions costs and of abstracting from holding period considerations.

C. Evidence on the Importance of Investor Heterogeneity

The high concentration of stock ownership in the U.S. supports the contention that frictions affect portfolio decisions, since it is easily shown that risk-averse agents with smooth preferences and positive wealth will hold at least a small portion of their wealth in stocks if stocks have a higher mean return than bonds (e.g., Haliassos and Bertaut (1991)). Limited market participation is often attributed to high fixed costs of entry (e.g., Saito (1991)), but it is hard to point to observable fixed costs which are large enough to explain the extent of non-participation observed in cross-sectional data. For instance, in the 1984 Panel Study of Income Dynamics (PSID), only 28% of the families surveyed owned any stock, and approximately 1/6 of these had holdings of less than $1,000. In the lower tail of wealth, 14 percent owned neither

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[23] The fact that the bond bears a coupon may make it more attractive to some institutionally constrained investors, but this appears unlikely to have a large affect on equilibrium prices.

stocks nor liquid assets [Mankiw and Zeldes (1990)].[24]

The paradox of non-participation is suggested by a simple calculation. Assume that a typical household anticipates annual savings of 6% of a $35,000 per year income between ages 30 and 65. Assume also that the mean real risk-free real rate is 3%, and the mean real return on stocks is 9%. Investing these savings entirely in bonds generates a yield of only $127,000 at age 65, while investing entirely in stocks has an expected yield of $453,000. Of course with high enough risk aversion this difference can represent a fair compensation for risk, but the magnitude of the difference makes the absence of stockholdings by so many households quite puzzling.

The observed concentration of stock ownership suggests that, at a minimum, the data used to test consumption-based models should reflect the consumption behavior of households that own at least some stock at some time. Unfortunately, obtaining disaggregated consumption data to use in these tests is difficult. For the U.S., one readily available proxy measure for the consumption of nondurables that is available along with information on stockholding is food consumption from the PSID. Mankiw and Zeldes (1990) compare the food consumption volatility of stockholders and nonstockholders, and calculate the correlation between consumption and asset returns for these two groups. They find that consumption volatility is higher for stockholders, and that consumption is more highly correlated with market returns. Conditioning on the consumption of stockholders, however, does not resolve the equity premium puzzle. Recalibrating the Mehra and Prescott (1985) model to match the observed equity premium requires a risk aversion coefficient of 35 for stockholders (versus 100 for all families in the PSID), suggesting that a representative wealthy agent model is not a solution to the equity premium puzzle.[25]

[24] Some have suggested that these statistics significantly understate stock ownership because they neglect indirect holdings through pension plans. However, Haliassos and Bertaut (1991) point out that the majority of pension plans have defined benefits, so that the policyholders still do not bear market risk. Furthermore, the overlap between stockholders and pensionholders is likely to be large since less wealthy households are less likely to have pensions.

[25] Mehra and Prescott consider 10 an upper bound on risk aversion, and many authors suggest that a coefficient of 1 or 2 is more reasonable. For a dissenting opinion, however, see Kocherlakota (1990).

D. Evidence on Consumption Volatility

If the models discussed in Section II are to reconcile the low risk-free rate and the equity premium, there are several key implications for consumption and income that should be observed: (a) individual consumption/income should be more volatile than aggregate consumption/income, (b) nondiversifiable income shocks should be autocorrelated, and (c) the conditional variance of individual income/consumption should increase in economic downturns. Evidence of borrowing or short-sales constraints also tends to support these models. Several recent studies using-cross-sectional data to examine how effectively households smooth consumption provide some insight on these issues.[26]

The evidence on borrowing constraints is mixed. Zeldes (1989) tests for the presence of liquidity constraints using the observation that the Lagrangian multiplier associated with the borrowing constraint of a constrained household will be positive, while that of an unconstrained household will be zero. Zeldes splits the sample between constrained and unconstrained households in the PSID, using other information to categorize each household. He concludes that there is evidence that constrained households face borrowing constraints. However Runkle (1991) finds little evidence of borrowing constraints using a different statistical methodology and a slightly different sample from the PSID. Cochrane (1991) uses a different approach based on the observation that if there were full insurance, then the correlation between consumption growth and a set of exogenous variables should be zero. Also using data from the PSID, he calculates the correlation between the food consumption growth rate and a set of exogenous variables that one might expect to affect consumption in the absence of complete insurance. He finds that full insurance is rejected for long illness and involuntary job loss, but not for spells of unemployment, loss of work due to strikes, or an involuntary move. These results seem loosely consistent with the idea that temporary shocks are smoothed by dissaving, but that permanent shocks affect consumption. McCarthy (1991) strengthens Cochrane's results by performing a similar exercise while controlling for wealth, and considering the role of non-financial wealth such as housing. In contrast, using data from the Consumer Expenditure Survey and a broader measure of consumption, Mace (1991) reports that she cannot reject complete insurance for a CARA utility specification, although she can for CRRA utility.

As discussed in Section II.A above, with incomplete insurance the persistence of income

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[26] Most of these studies were intended as tests of the permanent income or lifecycle savings hypotheses. Here we only discuss a small fraction of this extensive and closely related literature. For a more complete discussion see Attanasio (1994).

shocks will influence the degree of consumption smoothing. Carroll (1991) decomposes income innovations into a permanent and a transitory component and assumes that the shocks to the permanent and transitory components are independent. Under this identifying assumption, he finds a substantial role for permanent shocks and that the variance of the shocks to the permanent and transitory components are approximately the same. Heaton and Lucas (1994) estimate a different specification that allows for an interaction between aggregate and individual income innovations and also find evidence of significant persistence of individual shocks.

Whether the estimated persistence and volatility of individual income is large enough to explain the observed equity premium depends, among other things, on the assumed conditional variance of individual shocks over the business cycle. Heaton and Lucas (1994) find that the conditional variance of income increases in cyclical downturns, but the magnitude of the increase is too small to explain the equity premium in their model in the absence of transactions costs.

Finally, it should be noted that the greatest impediment to being able to make definitive statements about consumption and savings behavior is the paucity of data. For instance, one would like to look directly at the degree to which households use different categories of financial wealth to buffer income shocks over time, but there is little available individual or household data on financial assets with a signficant time dimension. Because of this, most of the studies discussed above take an indirect approach, and rely on the single observation of financial data collected in the 1984 PSID survey.

IV. Conclusions

Models with frictions in the form of borrowing constraints, transactions costs and undiversifiable risk have some intuitively attractive properties that make the traditional asset pricing puzzles less puzzling. In particular, for many of the different models that we have considered, calibration results suggest that the presence of small to moderate transactions costs, along with market incompleteness, can easily account for the low-risk free rate and in some cases, can explain the observed equity premium. This analysis also demonstrated the sometimes complex interaction between different types of frictions that can have ambiguous effects on predicted returns. For instance, we show that if income shocks have both a transitory and permanent component, more persistent shocks may either raise or lower equilibrium interest rates depending on the severity of the borrowing constraints. This sensitivity of model predictions to market structure can be viewed as undesireable in that it tends to preclude sharp empirical predictions. On the other hand, it has value in showing why more abstract models may fail when confronted with market data.

We also have surveyed some of the empirical evidence on the effects of market frictions. Although there are some encouraging results, the empirical importance of market frictions is still an open question. For example, it remains unclear whether observed market frictions are large enough to produce significant departures from the frictionless markets model. Further, the calibration results and much of the empirical research does not address the more difficult question of whether taking frictions explicitly into account can improve the predictive power of asset pricing models.

What implications might this have for future research? To better understand which implications of frictions are likely to be robust, more detailed modeling of the source of market frictions and their equilibrium effect on asset returns, portfolio policies, and holding periods would be useful. Perhaps, more important, better measurements of transactions costs, individual income processes and portfolio policies, etc., would greatly improve our ability to test these models, and might also point to new directions for building more realistic models of asset markets. For example, a notable difficulty with the models that we discuss is their inability to explain the heterogeneity in asset holdings across households. Apart from improving our understanding of asset prices, ultimately the models and empirical results that will be developed in response to these open issues should be useful in analyzing the welfare implications of various market structures and institutions.

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